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先渡換市場에서의 危險回避度에 관한 研究
〈국문요약〉

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선도환의 가격을 결정하는 접근방법에는 2차자산(derivative assets)이라는 선도계약의 기본특성에 기초한 재정거래(arbitrage)에 의한 방법이 가장 많이 이용되고 있다. 재정거래방식에는 선도환과 현물외환가격간의 상호관련성에 의하여 선도환가격을 이자율평가설(covered interest rate parity : CIRP), 즉 현물가격과 양국간의 이자율차이의 합으로 표시하고 있다. 특히 현물가격과 이자율은 모두 현재시점에서 의사결정자에게 알려져 있기 떄문에 선도환가격은 확실성하에서 결정되어 미래에 대한 예측이나 투자자의 위험회피도와는 관계없이 결정된다는 것이 특징이다. 이자율평가설에 관한 많은 실증연구는 거래비용을 고려한 경우 현실적으로 적절하다고 보고 있다(Frenkel and Levich ; 1975,1977).

다른 방법으로는 선도환의 미래예측기능에만 촛점을 맞추어 가격결정을 하는 투기. 예측접근방법(speculative efficiency approach : 이하에서는 SEA라 함)이 있다. 이 방법중에서 가장 단순한 형태로 표시된 가설, 즉 '선도환가격은 미래기대현물가격과 같다'는 가설은 대부분의 실증분석에서 기각되고 있다. 이에따라 SEA에서는 선도환가격이 미래에 대한 기대치뿐만 아니라 위험프리미엄까지 함께 포함하고 있다는 새로운 가설을 설정하고 이에대한 실증분석을 진행한다. 이 가설은 이론적 모형에서 출발한 것이 아니기 때문에, 특히 기대치와 위험프레미엄 모두가 측정불가능하다는 점으로 인하여 실증분석상 많은 어려움을 겪게 된다. 이러한 어려움을 피하기 위하여 많은 연구에서는 이자율평가설을 이용하여 선도환가격에 포함된 위험프레미엄에 대해 추론 내지 그 행태를 설명하려고 한다.

이자율평가설을 이용하여 분석모형을 설정하고 실증분석을 하는 것은 몇가지 근본적인 문제점을 내포하고 있다. 먼저, 앞서 지적한 바와 같이 이자율평가설을 가정한다는 것은 SEA에서 주된 관심이 되는 미래예측이나 위험프레미엄과는 관계없이 선도가격이 결정된다는 것을 의미한다. 따라서 이자율평가설을 가정하여 설정된 분석모형은 선도환시장의 효율성이나 균형가격결정에 대한 시사점을 제공할 수 없다는 것을 의미한다. 즉, 가정한

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시장효율성을 실증분석을 통하여 다시 검증하려는 것과 같다. 이러한 개념적 차원에서의 문제점 이외에도 실증분석에서의 추정상의 문제점 또한 존재한다. 대부분의 연구들이 현물자산의 균형가격결정모형에 이자율평가설을 추가로 결합하기 때문에 이러한 방법으로 설정한 분석모형은 그 기초가 되는 현물가격모형과는 달리 자의적 조작이 가능한 형태로 나타나며 이를 이용한 모수의 추정은 불필요한 편기(bias)를 가지게 된다.

본 연구에서는 이러한 실증분석상의 편기에 관한 문제점이 명확하고 구체적으로 나타나는 Mark(1985)의 실증연구를 재분석하고 실증자료를 통하여 위험회피도의 추정치에 편기가 발생하는 근본원인이 이자율평가설을 부적절하게 사용하는데 있다는 것을 확인하고자 한다. 실증분석결과는 본문의 〈표 1〉에 제시되어 있으며 그 내용을 간략하게 요약하면 다음과 같다.

(A) 실증분석모형 : 본 연구에서는 다기간 자산가격결정모형중에서 대표적인 Lucas (1978)모형을 직접 사용한다.

$$1=\beta\ E_t\left[\frac{U'(C_{t+1})\ P_t\ s_{t+1}}{U'(C_t)\ P_{t+1}\ s_t}\right] \tag{2}$$

$U'(c_t)$와 P_t는 t시점에서의 소비에 대한 한계효용과 소비재의 가격을, s_t와 f_t는 외환의 현물과 선도가격을, E_t와 β는 조건부 기대치와 시간할인계수를 나타낸다. Mark는 위의 식 (2)를 이자율평가설과 결합한 다음의 모형 (4)를 사용한다.

$$0=E_t\left[\frac{U'(C_{t+1})\ P_t(s_{t+1}-f_t)}{U'(C_t)\ P_{t+1}\ s_t}\right] \tag{4}$$

(B) 실증분석의 결과

위험회피계수 γ의 추정치 : Mark의 경우에는 γ의 추정치의 값이 0에서 50.38까지 매우 큰 폭의 변화를 보이고 있다. 특히 비내구성제품의 소비량과 선도프레미엄을 사용한 경우 γ 추정치의 값은 17.51로 비정상적으로 높게 나타난다. 반면에 본 연구에서는 추정치가 1.3으로 주식시장자료를 사용한 다른 연구결과와 비슷한 수준이다.〈Hansen and Sigleton (1982), Grossman and Shiller(1981), Friend and Blume(1975), Frankel(1983) 등을 참조〉

γ추정치의 정확도 : Mark에서는 추정치의 표준오차가 최소 15.65에서 최대 42.43 으로 매우 높은 반면 본 연구에서는 0.3에서 0.5수준으로 상대적으로 매우 정확한 추정결과를 보여주고 있다.

모형의 정확도 : 모형 (4)에 대한 적합도 검증은 사용된 도구변수(instrumental variables)의 종류에 따라 크게 차이가 난다. 시차변수(lagged variables)를 사용하지 않고 현

재소비와 선도프레미엄만을 사용할 경우 모형 (4)는 2.8% 또는 2.3% 유의수준에서 기각되는 반면 모형 (2)는 5% 유의수준에서 기각되지 않는다.

위와같은 실증분석의 결과는 앞서 논의한 바와 같이 이자율평가설을 사용하여 균형자산가격 결정모형을 변형시킴으로써 불필요한 편기를 발생시킨다는 것을 명확하게 보여주는 것이다.

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Risk Aversion in Forward Foreign Currency Markets

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I. Forward Contracts : Contingent Claims vs Predictors

There have been two approaches to the determination of the price of forward foreign currency. An arbitrage approach determines forward prices relative to spot prices and interest rates. The forward price is determined by imposing the zero profit condition on the arbitrage portfolio consisting of assets whose prices are currently known(i.e. the current spot price and interest rates). The price is thus given under certainty regardless of risk aversion and expectations of future states. In foreign currency markets, forward rates are given by the covered interest rate parity(CIRP). Frenkel and Levich (1975,1977) support CIRP with a correction for transaction costs, and some authors simply take it as granted. Mishkin (1985), for example, ascribes violation of CIRP to misaligned data by arguing that dealers in the foreign currency markets are setting prices according to or with a reference to the parity condition.[1)]

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1) See also Levich(1979, 1985) for the same assertion.

On the other hand, recent studies of forward exchange market efficiency, called the speculation approach[2], exclusively focus on the forecasting role of the forward. Since the forward price is directly related to unobservable market expectations of the future spot price, pricing forward contracts necessarily involves uncertainty. Viewing a forward as a predictor, statistical rejections of the unbiasedness hypothesis has been interpreted as 'speculative' inefficiency (Bilson : 1981) under risk neutrality, or the existence of the time-varying risk premium under risk aversion (Fama : 1984, Hansen and Hodrick : 1983, among others).

These two approaches (the predictor and arbitrage approaches) may represent two different ways of determining forward prices from different perspectives. That is, each approach may independently constitute its own distinctive hypothesis.[3] However, most of studies on the forward exchange speculation combine CIRP with the specific models of spot exchange rate in modelling forward biases or risk premia. The purpose of this study is to address econometric problems due to this combining procedure in implementing empirical tests under risk aversion.

The remainder of this study is organized as follows. We discuss in section 2 some conceptual problems in the speculation approach to motivate a further examination of some empirical studies in the literature. In section 3, we analyze the emirical work of Mark(1985) and show that his estimatio procedure yields biased results due to unnecessary use of CIRP. In the final section we briefly summarize the findings in empirical estimation and point out another conceptual problems implicit in the speculation approach for further reseachesin this area.

II. Forward Market Efficiency Under Risk Aversion

In the risk averse speculative efficiency approach, risk premia are defined in several different ways. They are either the expected profits of writing forward contracts since zero current investment is required in the forward transaction or they are the excess

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2) Writing forward contracts requires no net investment at the contracting time, hence the term 'speculation'.

3) Levich(1979, 1985) classifies CIRP as the test under certainty or riskfree, and the predictor hypothesis as under uncertainty respectively.

returns of the uncovered position over the forward covered position in foreign bonds markets.[4] More directly, the risk premium is modelled by combining the CIRP condition with the specific models of spot exchange rates. Fama(1984), for example, uses CIRP and the purchasing power parity condition and shows that the risk premium may be represented by the real interest rate differentials. Most of papers in the literature combine the intertemporal asset pricing model of Lucas(1978,1982) and CIRP.

In a single period context, Hansen and Hodrick(1983) consider "the difference between the two returns generated by buying foreign currency, investing in the risk-free, foreign-currency-denominated asset, and either leaving the proceeds uncovered or covering them in the forward market. This difference in returns, …… after some manipulation, satisfies" the following CAPM type model :

$$E_t \frac{S_{t+1} - f_t}{S_t} = \beta \ E_t [R_t^b - R_t] \tag{1}$$

where Et denotes the conditional expectation operator, st is the current spot rate, ft is one period forward exchange rates at t, Rtb is the rate of return on a bench-mark portfolio, and Rt is the rate of return on the zero beta portfolio.

Although Hansen and Hodrick use the percentage term normalized by st, and relate it to the rate of returns on portfolios, their approach defines only profits or returns of the forward speculation, not rates of returns. Because of the normalization problem, there is nothing to prevent the use of any currently known variables in place of st. Hansen and Hodrick(1983) normalize speculation profits by using current spot prices, but Hsieh and Kulatilaka (1982) use current forward prices. Roll and Solnik(1977) use CIRP condition to replace interest rate differentials by forward premia and write the international CAPM of Solnik(1973) essentially the same as (1). However, their model does not suffer from any arbitrariness since the model of Solnik(1973) is derived in terms of rates of returns on the spot market investment.[5]

4) See Hansen and Hodrick(1983) for these definitions.

5) See also Dusak(1973) and Black(1976) who write the CAPM with the percentage thanges in futures prices. Especially, Black notes that the rate of returns on futures can not be defined because of no current investment in futures contracting.

To model time varying property of risk premia, Hodrick and Srivastava (1984,1986), among others, use the intertemporal model of Lucas(1978,1982) and argue that the optimality condition must hold for both the covered and uncovered investments in a foreign currency markets. For the spot asset, the optimality condition is :

$$1=\beta E_t\left[\frac{U'(C_{t+1})\ P_t\ S_{t+1}}{U'(C_t)\ P_{t+1}\ S_t}\right] \tag{2}$$

The CIRP condition is :

$$1=\beta E_t\left[\frac{U'(C_{t+1})\ P_t}{U'(C_t)\ P_{t+1}}\right]\frac{f_t}{S_t} \tag{3}$$

where U'(ct) is the marginal utility of consumption at t, is the time discount factor, and pt is the currency price of the consumption goods at t. Taking difference between (2) and (3) and dividing by yields

$$0 = E_t\left[\frac{U'(C_{t+1})\ P_t\ (S_{t+1}-f_t)}{U'(C_t)\ P_{t+1}\ S_t}\right] \tag{4}$$

Note that, given the spot optimality condition (2), the CIRP equation (3) provides no additional information about the relationship between asset pricesand states of nature. Lucas(1982) shows within his complete market framework that the price of contingent contracts can be completely determined using the state prices and the current price of underlying spot asset. Hence the CIRP condition (3), written in terms of the marginal rate of substitution, simply defines the forward premium as the expectation or sum of the state prices, which is commonly denoted as the interest rates differential. Combining this redundant equation with the optimality condition (2) leaves a room for an arbitrary manipulation.

Since the final form of the model restriction (4) is set to zero as a consequence of combining the optimality condition with this redundant condition, manipulation with constants or any currently known variables becomes legitimate.

Multiplying both sides of (4) by any currently known variables does not affect the model restriction. In addition to the arbitrary normalizing variable problem, the time discount factor does not appear in (4). The marginal rate of substitution cannot be well defined because one can also divide by the current marginal utility of consumption.[6]

Above discussions indicate that problems arise from the misuse of the CIRP equation in relation to the exchange rate determination model. We address in the next section its econometric consequencies on the empirical estimation of risk premia in the context of Mark(1985). We show analytically and empirically that Mark's methodology creates unduly big biases in risk aversion coefficients estimates. We choose to investigate the work of Mark(1985) among many others, because Mark's model can be easily compared with the standard asset pricing model. Furthermore, Mark is the first to perform a specification test of a model on which the majority of studies on forward foreign exchange rates base their theoretical justification.

Ⅲ. Reestimation of Risk Aversion Coefficients

For the empirical estimation and testing of the model (4), Mark assumes a utility function of the form : $U(C) = \frac{C^r}{r}$ where $r = 1-\gamma$ and γ is the parameter of constant (non-negative) relative risk aversion. Using the instrumental variables estimation procedure of Hansen and Singleton(1982), Mark estimates the risk aversion parameter γ, and tests the restrictions (4). The findings are :

(a) Tests of the overidentifying restrictions[7] show weak evidence against the model restrictions (4). When instruments include the current and lagged values of the consumption ratio and speculation profits normalized by current spot rates, the restriction (4) is not rejected at the standard significance level for both measures of consumption (non-durable plus service and non-durable only). With the current consumption ratio and forward premium as instruments, the restriction is rejected at the significance levels of 2.8 percent and 2.3 percent when the consumption is measured by non-durable plus services and non-durable only respectively.

(b) Point estimates of risk aversion parameter γ and their correspondingstandard errors are quite large and vary substantially with the lags of instrument vectors and

6) See Hansen and Hodrick(1983), Mark(1985), Cumby(1985), Domowitz and Hakkio(1985), Hodrick and Srivastava(1983, 1986, 1987) among many others.

7) When there are m orthogonality conditions and n parameters to be estimated (m>n), m-n remaining conditions are not set to zero in estimation. But these overidentifying restrictions must be close to zero if the model is true.

with the measures of consumption.

The empirical results of Mark are in sharp contrast with those obtained by Hansen and Singleton(1982) who applied the same methodology to NYSE stock index data. In particular, point estimates of γ in Mark range from 0.00 to 50.38 and standard errors of estimates from 15.65 to 42.43, while estimates of Hansen-Singleton range from 0.68 to 0.97 and standard errors from 0.0629 to 0.3355. The point estimates of γ in Mark are unreasonably large.[8] Moreover, due to very large standard errors of estimates, it is difficult to make inferences about the effect of risk aversion, which is the main focus of the speculative approach under risk aversion as opposed to that under risk neutrality.[9] In other words, no reasonable hypothesis can be rejected, including the hypothesis of risk neutrality.

Mark ascribes these qualitative differences to the fact that Hansen and Singleton employ returns on a portfolio of assets, whereas Mark uses only individual asset returns. Although this may explain a part of the differences, there are more fundamental problems associated with Mark's approach. Deleting the time discount factor β, for example, may not be a theoretical problem, because is introduced into the model by a specific assumption on the intertemporal utility function. For empirical estimation and testing of the model restrictions, however, it creates biases in estimates of parameters because Mark's procedure introduces an additional unnecessary condition into the model and becomes composite hypotheses tests. To see this point, we note first that risk premia in forward exchange rates represent a conditional covariance between the marginal rate of substitution and real spot exchange rate. That is, risk premia can be completely characterized with two variables (the consumption and real spot exchange rate) and two parameters(β and γ). Mark, however, removes one parameter(β) in exchange for an additional assumption (CIRP) and a new variable (the forward price). As a result, the Mark's procedure becomes a test of composite hypotheses of

8) Grossman and Shiller(1981) use γ ranging from 0 to 4 for their analysis of the stock price variability. Estimates of Friend and Blume(1975) is about 2 when the only stochastic component of wealth is stock returns. In foreign currency markets, Frankel(1984) use the coefficient of 2. See also Dunn and Singleton(1987), and Rothemberg(1987).

9) Hansen and Singleton used 6 lagged variables as instruments, whereas Mark used 3 lagged variables. Since the standard error of estimates decreases as the number of instruments increases, this comparison is somewhat misleading.

two conditions (2) and (3).

Since the two equations (2) and (3) must hold simultaneously in order for Mark's model to hold, the model (2) is a necessary condition for equation (4) to hold. Mark's estimation procedure using the equation (4), therefore, must produce the same estimate of γ as one using equation (2) alone. But test results of Mark, in general, will not be the same as those based on condition (2) alone. Unless γ independent of β,[10] and CIRP holds exactly without error for every currency i and each time t, the parameter estimates of Mark and their corresponding standard errors are not the same as those of using (2) alone. A formal analysis is given in Appendix. Namely, the CIRP equation must be written without the expectation operator and hence the perfect foresight or certainty assumption about future consumption paths is required. This certainty assumption is much stronger than that implicit in his combining procedure : CIRP holds with expectations about consumption and price paths being rational on average. Furthermore, there is ample empirical evidence that CIRP does not hold exactly and continuously. The CIRP condition holds with a correction for transaction costs which may vary over time for empirical evidence. Given this empirical observation on CIRP, it is conceivable that Mark's estimation using (4) produces very poor estimates.

The above discussion suggests that using the optimality condition (2) alone may produce better estimation results than those of Mark without deviating from the main focal issues of Mark's study. Hence, we replicate tests based on (2) with the same set of data used in Mark to estimate risk aversion coefficients. Results are presented in Table 1, along with the reproduced results of Mark. Not surprisingly, the results show substantial differences in estimates and especially their corresponding standard errors. Estimates of risk aversion coefficients when the condition (2) is used are of reasonable size and are in line with those obtained by Hansen and Singleton(1982) and others. Moreover, the standard errors are relatively small so that all coefficients are significantly different from zero at a standard significance level. Tests of overidentifying restrictions cannot reject the model at 5 percent significance level. These results confirm all the predictions from discussions and analyses in the appendix.

10) This independence case is comparable to Mark's procedure since β does not appear in the equation (4).

Table 1 : Estimates of Risk Aversion Coefficients : Non-durable Consumption and Forward Premia are used as Instrumental Variables.

		Mark(1985)		Standard Intertemporal Model			
lag	d.f.	γ	Function	γ	Function	β	cov
0	19	17.51	33.227	1.3046	27.0799	0.9991	0.0015
		(26.25)	+	(0.4995)	* *	(0.0032)	
1	35	13.79	44.377	1.0205	41.9217	1.0007	0.0012
		(18.97)	*	(0.4369)	* *	(0.0029)	
2	51	12.67	57.079	0.4872	62.6898	0.0021	0.0006
		(16.64)	* *	(0.3043)	* *	(1.0031)	

Note : The standard errors are in parenthesis.
Function denotes the function value at its minimum.
cov denotes the covariance between β and γ
\+ denotes a marginal significance level of 2.5% under the null
* denotes a marginal significance level of 5.0% under the null
* * denotes a marginal significance level of 10.0% or more under the null.

Ⅳ. Concluding Remaks

We have addressed problems arising from the misuse of CIRP in relation to the models of the exchange rate determination. Focusing on its effects on the estimation of risk aversion coefficients, we have examined work of Mark(1985) to show that sources of estimation biases are closely related to the misuse of CIRP. Based on the standard intertemporal asset pricing model in place of Mark's combined equation, we have replicated empirical estimation and found evidence supporting our arguments.

Although we have discussed only specific econometric problems, we must also note that there are conceptual problems implicit in the simultaneous use of both the spot pricing model and CIRP. Unlike the arbitrage approach to forward pricing, the speculation approach exclusively focus on future expectations and risk premia implied in the forward price. However, once CIRP is assumed to hold, the parity-forward price is given by the current spot price and interest rates which are all known at the time of decision making. Thus the forward price itself is not all related to the expectation about future states of nature, nor to the degree of investors' risk aversion. Phaup (1981) and Levi(1984) show that regardless of the expectation of investors, the forward price must be determined by CIRP when investors are allowed to take a long (short) position in spot currencies and a corresponding simultaneous short(long) position in bonds.[11] Hence, the assumed CIRP condition becomes incompatible with the predictor hypothesis being studied.

As indicated by above discussions, the arbitrage condition(CIRP) and the speculation approach are not complementary to each other. Rather, they are in general mutually exclusive. Assuming the CIRP condition hold is therefore equivalent to assuming the forward market is efficient according to the arbitrage approach, and the parity-forward price is the equilibrium price. Modelling the risk premium with CIRP and testing the speculation hypothesis is equivalent to testing the assumed efficiency of the forward market again. Hence, this approach bears no implications for the forward market efficiency or the determination of forward prices.

Once the equilibrium spot models are specified, the assumed CIRP equation defines interest rates differentials as forward premia. Thus forward premia may replace interest rates differentials in testing spot models or exchange risk. Cumby and Obstfeld (1984) note that the unbiased predictor hypothesis becomes identical to the Fisher open hypothesis or uncovered interest parity, which is the simplest form of spot exchange models or a direct representation of spot speculation. Thus, the speculation approach may be regarded as an indirect way of testing spot models. However, modelling risk premia directly from the predictor hypothesis suffers from the arbitrary manipulation problem.

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11) The speculation approach exclusively deals with forward market speculations only, although spot market speculations are more common and natural ways of describing investment behaviors. As long as there is a dependent relationship between spot and forward prices(the CIRP condition), however, each of two speculations should provide the same implications for market behaviors.

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Appendix : Analysis of Estimation Bias

Instrumental Variables Estimation of Hansen and Singleton(1982) :

Consider a problem of estimating parameters b (n dimensional) in a function : $E_t h(x_{t+1}, b) = 0$, where h(.) is the $(m \times 1)$ vector function $(m > n)$; and x_{t+1} is a m dimensional vector of variables observed at t+1. Let z_t denote a q dimensional vector of instrumental variables in agents' information set. Assume x_{t+1} and z_t have finite second moments. Define functions k and g by

$$k(x_{t+1}, z_t, b) = h(x_{t+1}, b) \otimes z_t$$

$$g(b) = E[k(x_{t+1}, z_t, b)]$$

where $\otimes$ denotes the Kronecker product and E is the unconditional expectation operator. Denote the true values of parameters by b_0. Then $g(b_0) = 0$ since z_t is a subset of agents' information set. If the underlying model is true, then the method of moment estimator of the function g(b), $g_T(b)$,

$$g_T(b) = \frac{1}{T} \sum_{t=1}^{T} k(x_{t+1}, z_t, b)$$

evaluated at $b = b_0$ should be close to zero for large values of T.

Hansen(1982) proposes that b be chosen to minimize the 'distance function'

$$J_T(b) = g_T(b)' W_T \, g_T \tag{A1}$$

where $W_t = [\frac{1}{T} \sum_{t=1}^{T} k(x_{t+1}, z_t, b) \; k(x_{t+1}, z_t, b)']^{-1}$

The first order condition from the minimization of (A1) is

$$\frac{\partial g_T(b)'}{\partial b} W_T \, g_T(b) = 0 \tag{A2}$$

Let $\hat{b}$ be the minimizer of (A1). In large sample, $\hat{T}(b\text{-}b)$ is distributed as a normal with mean

0, and covariance matrix $(d_T\ W_T\ d_T)^{-1}$, where

$$d_T = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial k(x_{t+1},\ z_t,\ b)}{\partial b}$$

Since T times J_T at minimum has a chi-square distribution with (mq-n) degrees of freedom, it can be used to test the overidentifying restrictions.

Comparisons of Estimates :

For simplicity, we consider a special case in which $h(x_{t+1},b)$ is a sequence of independent, random vectors and $h(x_{t+1},b)$ is independent of z_t. That is,

$$E[h(x_{t+1},b)\ \mid\ z_t,\ h(x_t,b),\ z_{t-1},\ \cdots\cdots] = 0$$
$$E[h(x_{t+1},b)\ h(x_{t+1},b)'\ \mid\ z_t,\ h(x_t,b),\ z_{t-1},\ \cdots\cdots] = R$$

Let Z_t denote a vector of instrumental variables ; $Z_t = \{z_t,\ z_{t-1},\ \cdots\cdots\}$.
Then $W_T = [R \otimes Q]^{-1}$, where $Q = E(Z_t Z_t\ )$. It is further assumed that R is diagonal, $Diag(R) = \{R_i\}$, $i = 1, \cdots, m$.

In Mark's procedure, $(m \times 1)$ vector function G(.) is given by

$$G(x_{t+1},\ r) = E[h(x_{t+1},\ b)] = \{E\ C_{t+1}^{-\gamma}\ (S_{t+1,\ i} - F_{t,i})\},\ i = 1, \cdots\ m.$$

where $C_{t+1} = c_{t+1}/c_t$, $S_{t+1,i} = s_{t+1,i}/s_{t,i}$, $F_{t,i} = f_{t,i}/s_{t,i}$, and the subscript i denotes the i^{th} currency. For notational convenience, it is assumed that there is no inflation ; i.e. $p_t = 1$ for all t. The first order condition (A2) with respect to γ is

$$G_o'\ R^{-1}\ G \times Z_t'\ Q^{-1}\ Z_t = O,$$

$$G_o = \frac{\partial G}{\partial \gamma} = \{\ E[C_{t+1}^{-\gamma}\ \log(C_{t+1})\ \ (S_{t+1,\ i} - F_{t,i})]\ \},\ i = 1,\ \cdots\cdots,\ m$$

$$G_o'R^{-1}G = \sum_{i=0}^{m} E[(S_{t+1,i} - F_{t,i})\ C_{t+1}^{-\gamma}\ \log(C_{t+1})]\ E[(s_{t+1,i} - F_{t,i})\ C_{t+1}^{-\gamma}\ ]/R_i \quad (A3)$$

$$R_i = E[\ C_{t+1}^{-\gamma}\ (S_{t+1,i} - F_{t,i})]^2 \quad (A4)$$

When the optimality condition (3.2) alone is used, the $(m\times 1)$ vector function $H(.)$, corresponding to $G(.)$ of Mark is given by

$$H(x_{t+1}, b) = \{\beta\ E[C_{t+1}^{-\gamma}\ S_{t+1,i}] - 1\},\ i = 1, \cdots, m$$

where $b = (\gamma, \beta)$. The first order condition with respect to γ is

$$H_o'R_H^{-1}H \otimes Z_t'Q^{-1}\ Z_t = 0,$$

$$H_o = \frac{\partial H}{\partial \gamma} = \{\beta E\, C_{t+1}^{-\gamma} \log(C_{t+1})\ S_{t+1,i}\},\ i = 1, \cdots\cdots, m$$

$$H_o'\ R_H^{-1}H = \beta\Sigma_{i=1}^{\mu}\ E[S_{t+1,i}\ C_{t+1}^{-\gamma} \log(C_{t+1})]\ E[\beta S_{t+1,i}\ C_{t+1}^{-\gamma} - 1]/R_{H,i} \tag{A5}$$

$$R_{H,i} = \frac{\partial G}{\partial \gamma} = [\beta C_{t+1}^{-\gamma}\ S_{t+1,i} - 1]^2 \tag{A6}$$

Estimates of γ from two procedures differ unless (A3) coincides with (A5), which can be true only if CIRP condition is written without an expectation operator (certainty about future consumption paths). The CIRP condition can be written as : $\beta F_{t,i} C_{t+1}^{-\gamma} = 1 + \beta F_{t,i}\ \varepsilon_{t+1}$, because $C_{t+1}^{-\gamma} = E_t C_{t+1}^{-\gamma} + \varepsilon_{t+1}$, where ε_{t+1} is the raional forecast error. Hence $R_{H,i} = \beta^2\ R_i$, only if $F_{t,i}\ \varepsilon_{t+1}$ is identically zero for all i and t, which can be true under certainty or perfect foresight.[1] When $R_{H,i} = \beta^2 R_i$ the deviation can be written as :

$$G_o'R^{-1}G - H_o'R_H^{-1}H = -\Sigma_{i=1}^{m}\ E[F_{t,i} C_{t+1}^{-\gamma} \log(C_{t+1})]\ E[S_{t+1,i} - F_{t,i}) C_{t+1}^{-\gamma}\ ]/R_i \tag{A7}$$

If the second term in the numerator is zero for all i, then the estimation becomes degenerate $(G_o'R_H^{-1}G = H_o'R_H^{-1}H = 0)$. The first term becomes zero, $[E\{F_{t,i}C_{t+1}^{-\gamma}\ \log(C_{t+1})\} = 0]$ from the first derivative of the CIRP equation (3.3) with respect to γ. This clearly shows the bias due to unnecessarily introduced assumption : CIRP written in terms of expected future consumption paths. Mark's procedure is equivalent to taking derivatives of the equations (3.2) and (3.3) with respect to the same parameter γ and then combine two resulting conditions.

Unless the CIRP equation holds with certainty about future consumption paths, there is no guarantee that this term be zero for all i. Since Mark implicitly assumes CIRP is satisfied on average, this term need not be identically zero so that this additional term is a source of estimation biases.

Estimation biases can be magnified in the standard error of γ estimates. For Mark's procedure, the standard error of the γ estimate is the squared root of $(G_0'R^{-1}G_0)^{-1} \times (Z_t'Q^{-1}Z_t)^{-1}$

For the procedure using (3.2) alone when estimates of γ and β are independent (diagonal information matrix), the standard error of the γ estimate is the squared root of $(G_o'R_H^{-1}H_o)^{-1} \otimes (Z_t'Q^{-1}Zt)^{-1}$.

The difference, when $R_{H,i} = R_i$, is given by

$$G_o'R^{-1}G_o - H_o'R_H^{-1}H_o = \Sigma_{i=1}^{m} \{EF_{t,i}\ C_{t+1}^{-\gamma} \log(C_{t+1})\}^2 / R_i$$

$$-2\Sigma_{i=1}^{\mu} \{EF_{t,i}\ C_{t+1}^{-\gamma} \log(C_{t+1})\}\ \{ES_{t+1,i} C_{t+1}^{-\gamma} \log(C_{t+1})\} / R_i$$

The terms in the right hand side represent the bias introduced by Mark's combining procedure. The bias becomes very large, especially when the spot model (3.2) is true so that the second term is zero.

In addition to estimation biases discussed above, the combining procedure of Mark may possibly explain somewhat peculiar findings on the tests of the overidentifying restrictions ; th findings that the model is rejected only when current forward premia, $F_{t,i}$, are used as instrumental variables. When $Z_t = \{F_{t,i}\}$, the 'distance function', J_T involves $E[C_{t+1}^{-\gamma} (S_{t+1,i} - F_{t,i})\ F_{t,j}]$, for Mark's procedure. Unless the CIRP condition holds without forecast errors of future consumption and price paths such that we can substitute $C_{t+1}^{-\gamma} F_{t,i}$ for a constant for all i, the 'distance functions' for both procedures can not be identical. It is therefore possible that Mark's tests reject the orthogonality restrictions even when the underlying model is true and CIRP holds. In his actual estimation, Mark uses the terms $E[F_{t,i}F_{t,j} C_{t+1}^{-\gamma}]$. Although $E[F_{t,k} C_{t+1}^{-\gamma}]$ is or close to a constant if CIRP holds on average, this does not ensure that the cross product terms, $E[F_{t,i}F_{t,j} C_{t+1}^{-\gamma}$ reduce to a constant multiple of $F_{t,i}$. Due to these squared terms and frequently observed heteroscedacity, it is more likely that tests of Mark reject the model restrictions.

1) With CRIP holding on average, $E\{F_{t,i}\ \varepsilon_{t+1}\}=0$, implicit in Mark's combining procedure.